Eliassen–Palm (EP) flux

1. Can be easily derived from $\overline{v'q'}$

$$q'(x,y,z,t) = \nabla^2\psi' + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi'}{\partial z}\right).$$

$b' = f_0\partial\psi'/\partial z$

Rearrange into the form of divergence

$$v'q' = -\frac{\partial}{\partial y}(u'v') + \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}v'b'\right) + \frac{1}{2}\frac{\partial}{\partial x}\left((v'^2 - u'^2) - \frac{b'^2}{N^2}\right).$$

Zonal average

$$\overline{v'q'} = -\frac{\partial}{\partial y}\overline{u'v'} + \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}\overline{v'b'}\right).$$

$\overline{v'q'} = \nabla_x \cdot \boldsymbol{\mathcal{F}},$

$$\boldsymbol{\mathcal{F}} \equiv -\overline{u'v'}\,\mathbf{j} + \frac{f_0}{N^2}\overline{v'b'}\,\mathbf{k}$$

2. The Eliassen–Palm relation

$$\boldsymbol{\mathcal{F}} \equiv -\overline{u'v'}\,\mathbf{j} + \frac{f_0}{N^2}\overline{v'b'}\,\mathbf{k}$$

$$\frac{\partial \mathcal{A}}{\partial t} + \nabla \cdot \boldsymbol{\mathcal{F}} = \cancel{\mathcal{D}},$$

$$q' \times \left(\frac{\partial q'}{\partial t} + \overline{u}\frac{\partial q'}{\partial x} + v'\frac{\partial \overline{q}}{\partial y} = \cancel{D'}, \right) \longrightarrow \frac{1}{2}\frac{\partial}{\partial t}\overline{q'^2} = -\overline{v'q'}\frac{\partial \overline{q}}{\partial y} + \cancel{\overline{D'q'}}.$$

$\partial\overline{q}/\partial y$ independent of t

$$\mathcal{A} = \frac{\overline{q'^2}}{2\partial\overline{q}/\partial y}, \qquad \mathcal{D} = \cancel{\frac{\overline{D'q'}}{\partial\overline{q}/\partial y}},$$

For plane waves $F = cA$

With group velocity of Rossby waves

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial \mathcal{A}}{\partial t} + \nabla \cdot (\mathcal{A}\mathbf{c}_g) = 0.$$

Wave activity density 波作用量守恒 $\frac{\mathrm{d}}{\mathrm{d}t}\int_A \mathcal{A}\,\mathrm{d}A = 0.$

$$\mathscr{E} = K^2 A_0^2/4, \qquad \frac{\partial \mathscr{E}}{\partial T} + \nabla \cdot \mathscr{E}\mathbf{c}_g = \frac{1}{4}A_0^2\left(2kl\frac{\partial \bar{u}}{\partial Y} + 2\frac{f_0^2}{N^2}kn\frac{\partial \bar{u}}{\partial Z}\right).$$

$\overline{u}$ 变化时 Rossby 波波能密度不守恒

3. To understand $v'b'$

3.1 Buoyancy, stream function and thermal wind balance

Thermal wind balance we have learnt:

$$\frac{\partial \overrightarrow{v_g}}{\partial z} = -\frac{1}{f}\frac{\partial}{\partial z}\left(-\frac{1}{\rho_s}\nabla_h P'\right) \times \vec{k}$$

Vertical part of QG eqations $\frac{1}{\rho_0}\frac{\partial P'}{\partial z} = \frac{\rho'}{\rho_s} g$

Geostrophic stream function $\psi = \frac{P'}{f_0 \rho_s}$

$$b = f_0 \frac{\partial \psi}{\partial z},$$

$f_0 \frac{\partial \psi}{\partial z} = \frac{\partial}{\partial z}\left(\frac{P'}{\rho_s}\right) = g\frac{\theta'}{\theta_s} = b$ (Boussinesq defination)

Geostrophic wind $u = -\frac{\partial \psi}{\partial y}$

Zonal mean $\bar{\psi} = \bar{\psi}(y; z)$

$$f_0 \frac{\partial \bar{u}}{\partial z} = -\frac{\partial \bar{b}}{\partial y},$$

$$\frac{\partial^2 \bar{\psi}}{\partial y \partial z} = f_0 \frac{\partial \bar{u}}{\partial z} = -\frac{\partial \bar{b}}{\partial y}$$

3.2 QGPV and E-P flux in different forms

Vallis

$$\boldsymbol{\mathcal{F}} \equiv -\overline{u'v'}\,\mathbf{j} + \frac{f_0}{N^2}\overline{v'b'}\,\mathbf{k}$$

$$q = \beta y + \left[\nabla^2 + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial}{\partial z}\right)\right]\psi.$$

$$q = \beta y + \zeta + \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}b\right)$$

刘式适

$$\overline{q'v'} = -\frac{\partial}{\partial y}\overline{u'v'} + \frac{f_0}{\partial\theta_0/\partial z}\frac{\partial}{\partial z}\overline{\theta'v'}$$

Holton

$$F_y = -\rho_0\overline{u'v'}, \quad F_z = \rho_0 f_0 R\overline{v'T'}/(N^2H)$$

$$\overline{q'v'} = -\frac{\partial\overline{u'v'}}{\partial y} + \frac{f_0}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\rho_0}{N^2}\overline{v'\frac{\partial\Phi'}{\partial z}}\right)$$

$$\overline{q} = f_0 + \beta y + \frac{1}{f_0}\frac{\partial^2\overline{\Phi}}{\partial y^2} + \frac{f_0}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\rho_0}{N^2}\frac{\partial\overline{\Phi}}{\partial z}\right) \qquad \psi \equiv \frac{\phi}{f_0}$$

3.3 Understand terms in EP flux

$$\boldsymbol{\mathcal{F}} \equiv -\overline{u'v'}\,\mathbf{j} + \frac{f_0}{N^2}\overline{v'b'}\,\mathbf{k}$$

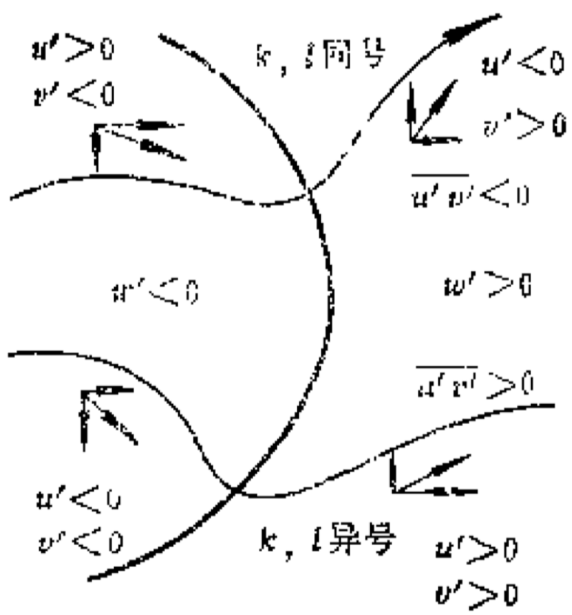
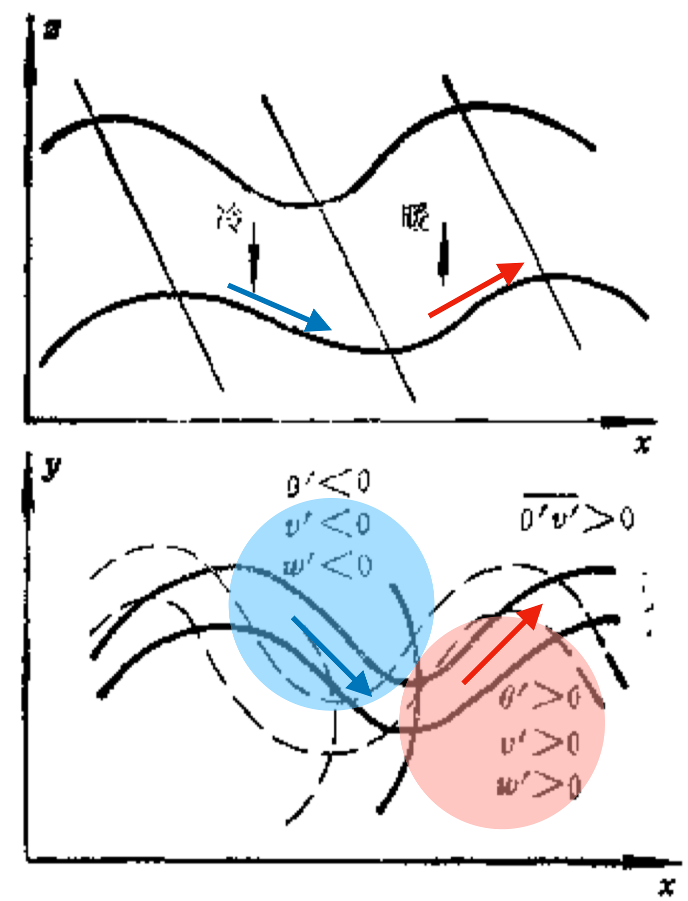


图8.3 Rossby 波对动量的经向输送

脊槽线自下而上向西倾斜



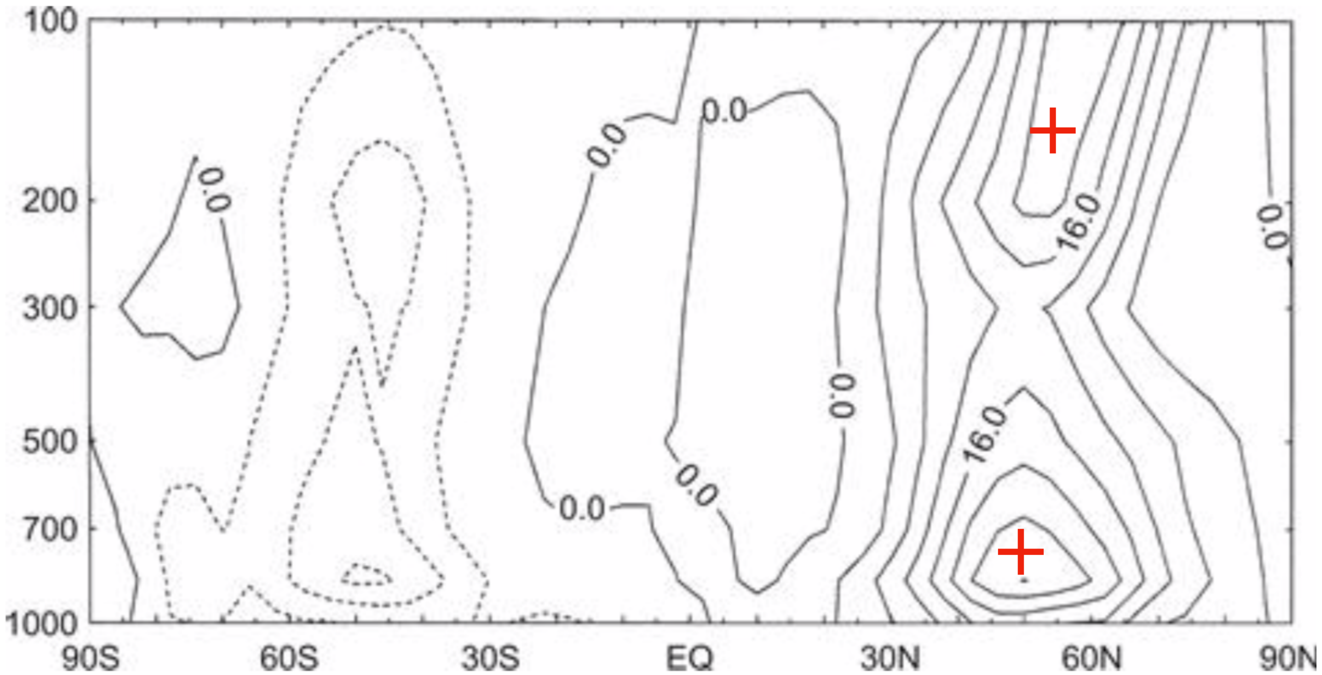
垂直剖面

水平面

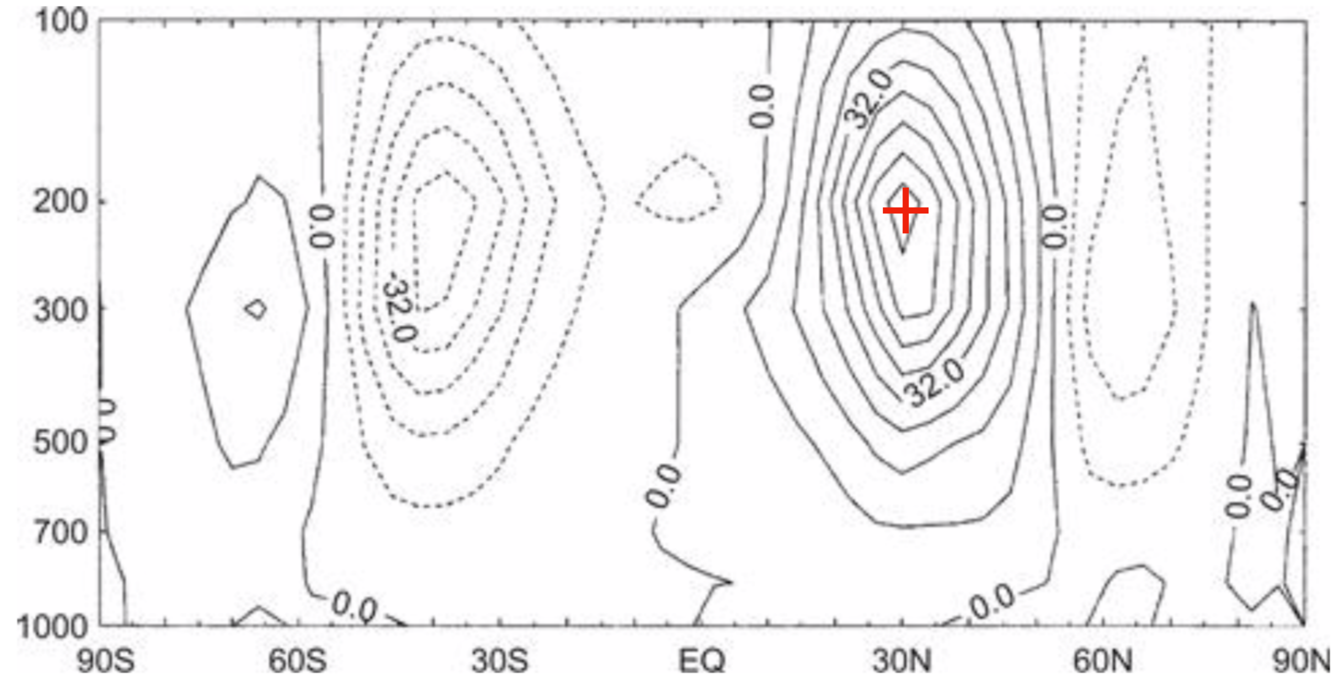
图8.4 Rossby 波对热量的经向输送

4. Distribution of EP flux and its terms

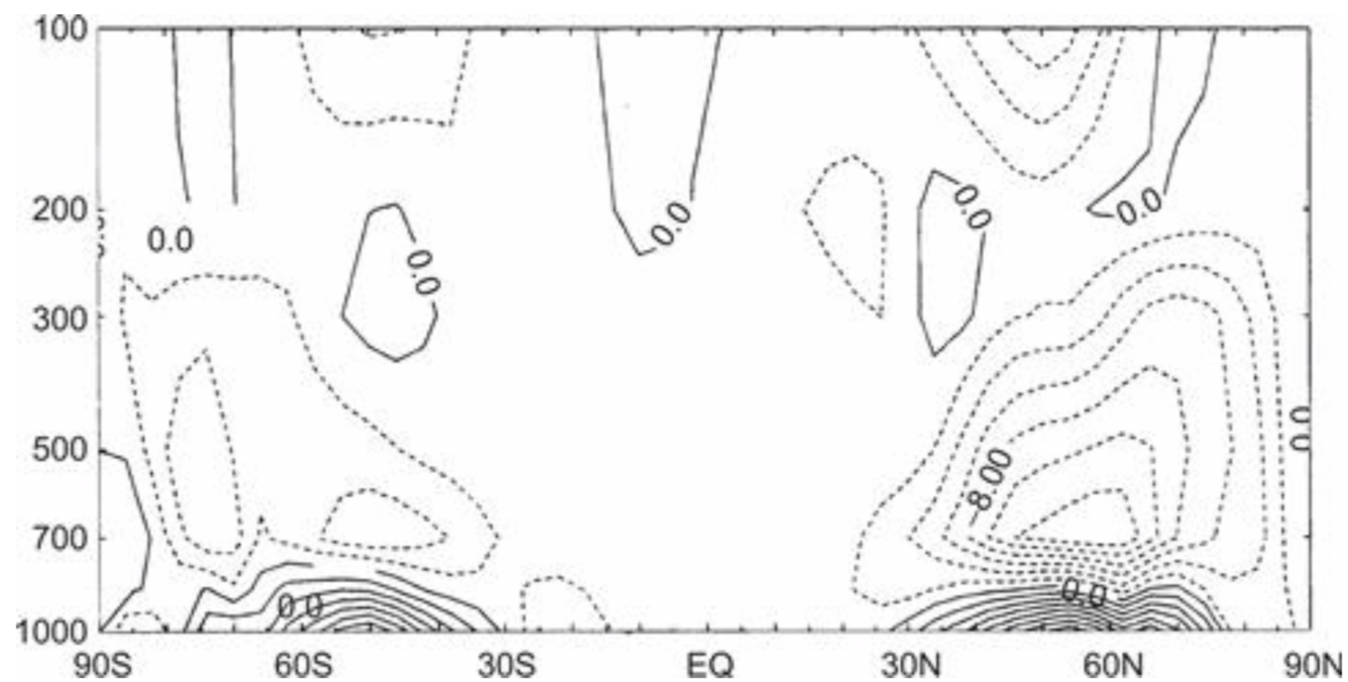
eddy heat flux



eddy momentum flux



Eliassen–Palm flux divergence



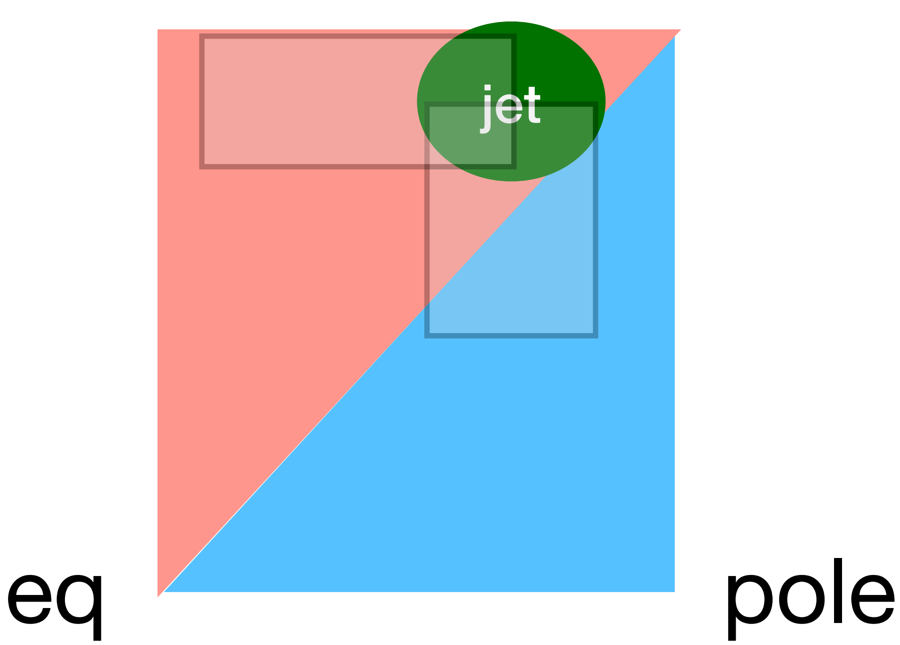
5. From EP flux to mean flow

5.1 Physical meaning

Zonally-averaged Eulerian Boussinesq equations with QG scaling

动量输送改变 u	$\partial\overline{u}/\partial t - f_0\overline{v} = -\partial\left(\overline{u'v'}\right)/\partial y + \overline{X}$	$\frac{\partial \bar{u}}{\partial t} = f_0\bar{v} - \frac{\partial}{\partial y}\overline{u'v'} + \overline{F},$
热量输送改变 T	$\partial\overline{T}/\partial t + N^2HR^{-1}\overline{w} = -\partial\left(\overline{v'T'}\right)/\partial y + \overline{J}/c_p$	$\frac{\partial \bar{b}}{\partial t} = -N^2\bar{w} - \frac{\partial}{\partial y}\overline{v'b'} + \overline{S}.$
热成风关系	$f_0\partial\overline{u}/\partial z + RH^{-1}\partial\overline{T}/\partial y = 0$	$f_0\frac{\partial \bar{u}}{\partial z} = -\frac{\partial \bar{b}}{\partial y},$

$\overline{v'b'} \longrightarrow \bar{u}$



中纬度对流层上层$\left(\overline{v'T'}\right)_{max}$

下方$\frac{\partial\left(\overline{v'T'}\right)}{\partial z} > 0$，暖侧$\frac{\partial\left(\overline{v'T'}\right)}{\partial y} > 0$

假设$\overline{v'T'}$有一正扰动，

- 暖侧$\frac{\partial\left(\overline{v'T'}\right)}{\partial y} > 0$，根据上面的方程，$\frac{\partial \bar{T}}{\partial t} < 0$，暖侧发生冷却

  冷却后$\frac{\partial \bar{T}}{\partial y}$减小，根据热成风关系$\frac{\partial \bar{u}}{\partial z}$增加，即急流加强

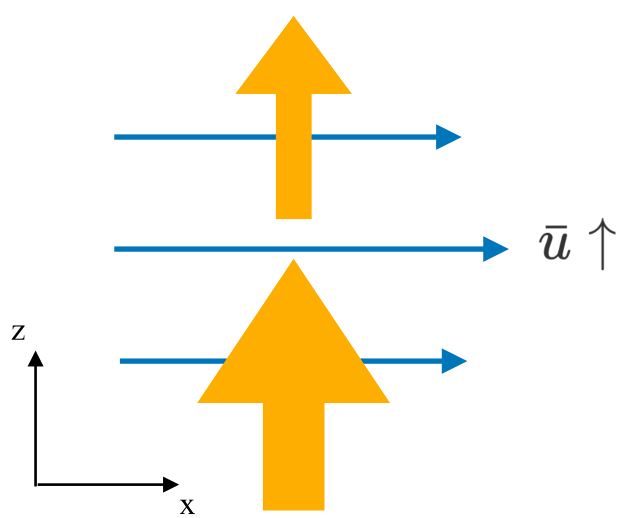
- 下方$\frac{\partial\left(\overline{v'T'}\right)}{\partial z}$增加，EP flux 散度为正，急流加强

5.1 Physical meaning

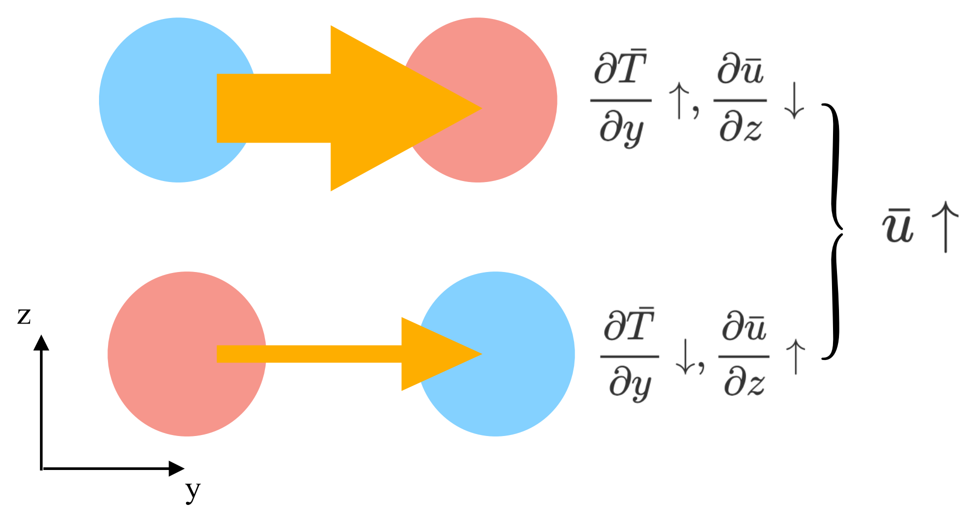
$$\overline{v'q'} = -\frac{\partial}{\partial y}\overline{u'v'} + \frac{\partial}{\partial z}\left(\frac{f_0}{N^2}\overline{v'b'}\right).$$

$$f_0\frac{\partial \overline{u}}{\partial z} = -\frac{\partial \overline{b}}{\partial y},$$

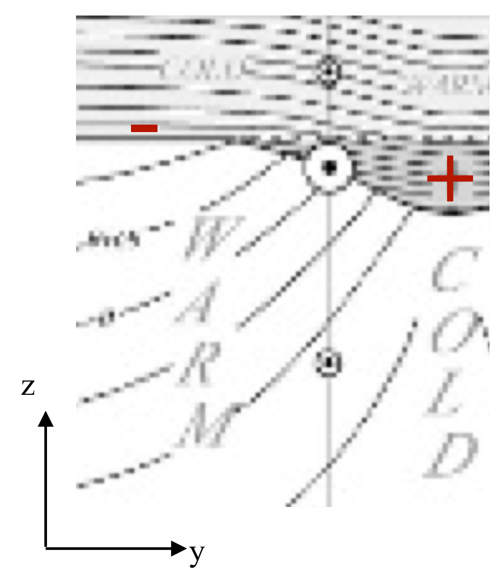
$$\frac{\partial \overline{u'v'}}{\partial y} < 0$$



$$\frac{\partial \overline{v'T'}}{\partial z} > 0$$



$$\overline{v'q'} > 0$$



5.2 Mathematical aspect

PV inversion $$q = \beta y + \left[\nabla^2 + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial}{\partial z}\right)\right]\psi.$$

(change of…)

$$\overline{v'q'} \longrightarrow \bar{q} \longrightarrow \bar{\psi} \longrightarrow \bar{u}$$

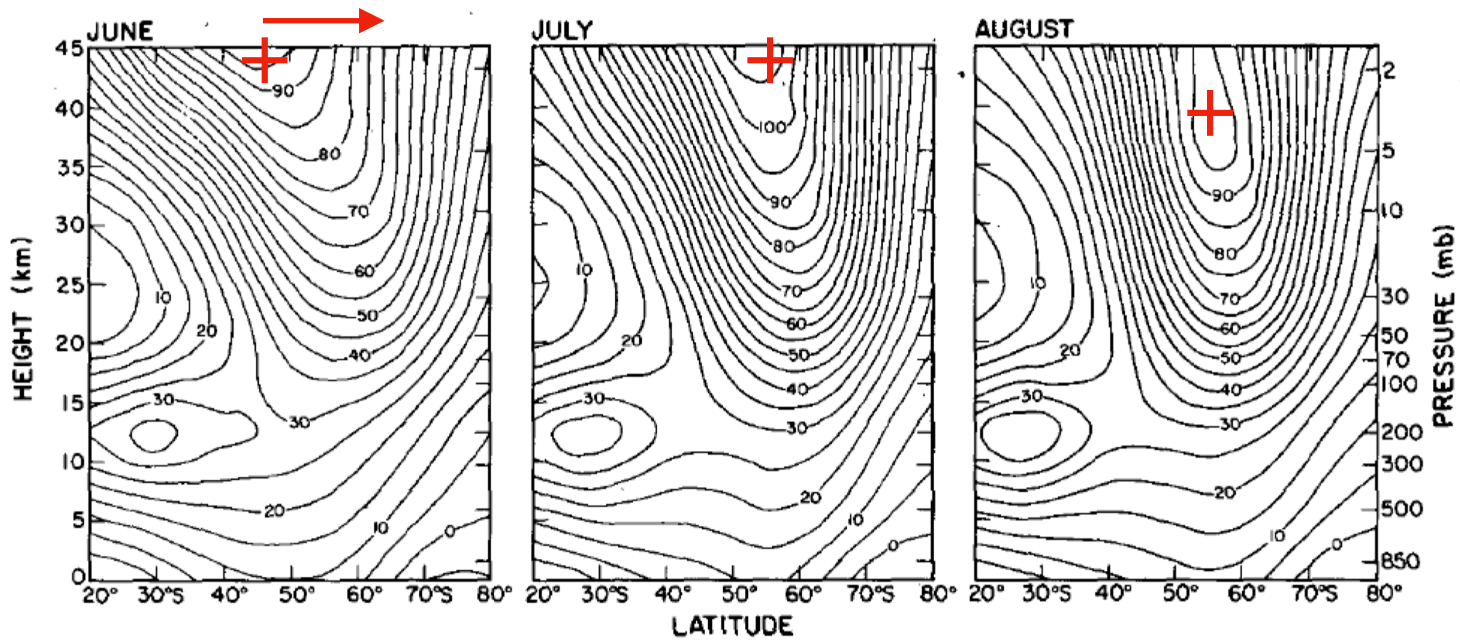
5.3 Equations to link them

transformed eulerian mean (TEM)

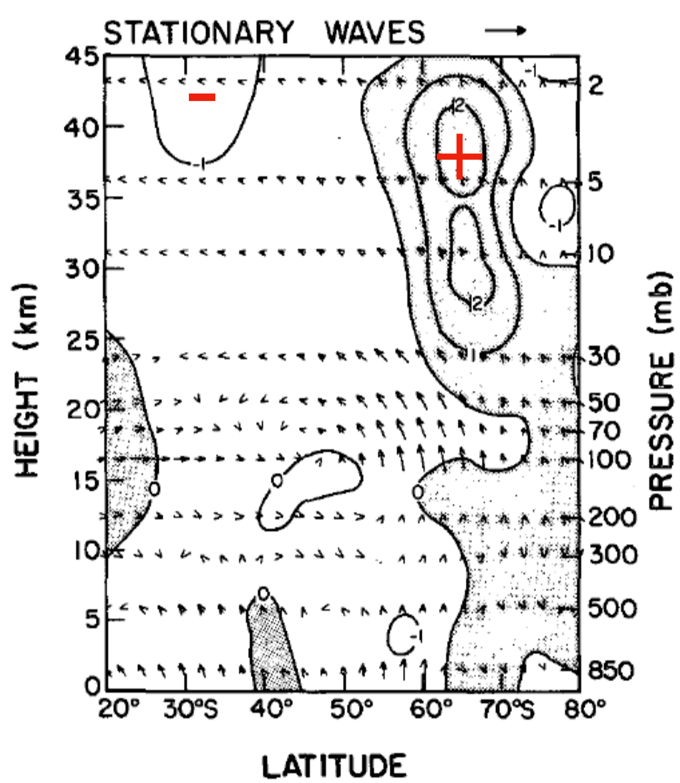
$$\frac{\partial \overline{u}}{\partial t} = f_0 \overline{v}^* + \overline{v'q'} + \overline{F},$$

$$\frac{\partial \overline{b}}{\partial t} = -N^2 \overline{w}^* + \overline{S},$$

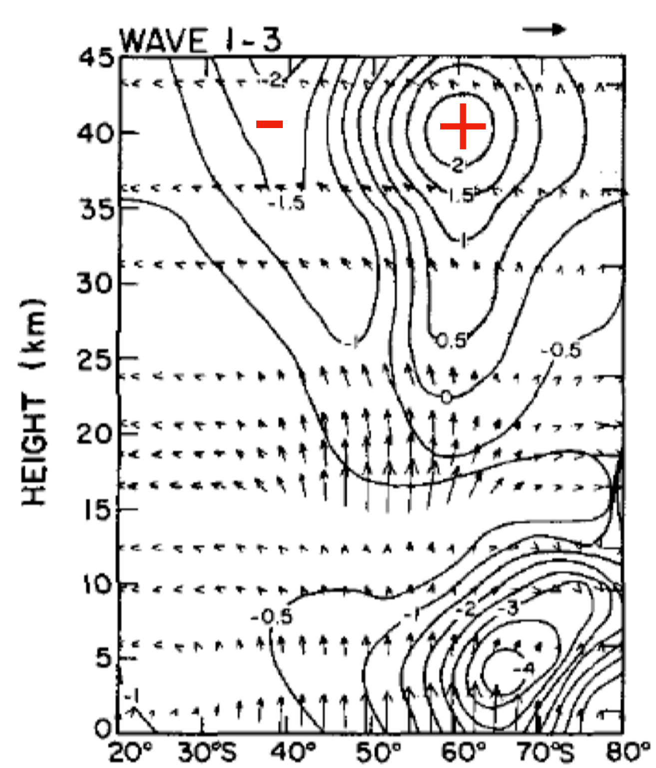
6. An observation of wave-mean flow interaction



Zonal wave number



Stationary



transient

Hartmann et al, 1984

7. More realistic forms

Spherical coordinates and moist effect

Hartmann et al, 1984

$$F_{(\phi)} = -a\cos(\phi)\overline{v'u'} \quad F_{(p)} = a\cos(\phi) f\, \overline{v'(\theta' + h'L/c_p} / \overline{\theta}_p$$

Dwyer and O'Gorman, 2017

$$F^{(\phi)} = -a\cos\phi\overline{u'v'}$$

$$F^{(p)} = a\cos\phi f\frac{\overline{v'\theta'}}{\overline{\theta}_p}.$$

→

(1) $$-\left(\frac{\partial\theta}{\partial p}\right)_{\text{eff}} = -\frac{\partial\overline{\theta}}{\partial p} + \lambda\left.\frac{\partial\theta}{\partial p}\right|_{\theta*}$$

(2) $$\theta \rightarrow \theta_e$$

$$\theta_e = T_e\left(\frac{p_0}{p}\right)^{\kappa_d} \approx \left(T + \frac{L_v}{c_{pd}}r\right)\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_{pd}}}$$

Thank you